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# Trajectory Planning of Spherical Pendulum Pattern for Application in Creating Batik Patterns

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***Abstract*— Batik Pendulum is a new batik pattern created by Rumah Batik Komar using a single-string pendulum filled with wax. However, current production is still manual, so it is impossible to re-manufacture in large quantities. This research is part of a machine and software development project to produce Batik Pendulum, where this research focuses on software development. The designed software has a spherical pendulum trajectory planning feature through parameter changes. The spherical pendulum path was chosen because it has the same pattern as the currently produced Batik Pendulum. An algorithm that receives parameter values input has been designed to produce the spherical pendulum trajectory. These proposed parameters provide a variety of spherical pendulum patterns. After the user changes the parameters, the software requires 1-2 seconds to generate the trajectory. This duration is within the user's flow of thought when engaging with the software.**

***Index Terms*— batik patterns; batik pendulum software; batik trajectory planning; spherical pendulum trajectory planning; trajectory planning software.**

## I. Introduction

Batik is a traditional Indonesian cloth designated by UNESCO as a masterpiece of the oral and intangible heritage of humanity on October 2, 2009. Batik is a detailed patterned textile made through a repetitive process, namely drawing or stamping wax on the fabric, dyeing, removing wax, washing, drying, and repeating until the desired pattern and color is achieved. The uniqueness of batik lies in its pattern. Batik has two basic patterns, geometric and non-geometric, each with characteristics [1]. Most traditional patterns are drawn by hand. Along with the growth of computer algorithms for generating patterns, contemporary batik patterns, such as fractals, have recently been generated using software and fractal algorithms [2] and then hand-drawn on fabric and proceeding to the batik production procedure.

The batik production process occurs in eight provinces in Indonesia—Jambi, Bengkulu, Jakarta, West Java, Central Java, Yogyakarta, East Java, and Bali—which produce the majority of the country's batik [3]. However, due to social distancing health policies, the COVID-19 pandemic disrupted this business by laying off most of the workforce. Therefore, Batik Small and Medium Enterprises (SMEs) must innovate to respond to this labor shortage [4]. Automating a part of the production line is one way to compensate for the shortage of batik production workers.

(a) (b)

Fig. 1. (a) Single-string pendulum [5] & (b) Batik Pendulum product

Rumah Batik Komar, a batik business in West Java, Indonesia, is thriving in creative patterns and automation of production processes. The latest innovation is the Batik Pendulum [5], which produces unique batik patterns using a single-string pendulum filled with wax. The pendulum is swung from a certain angle and then released to oscillate naturally. Competent craftsmen operate this pendulum to produce artistic circular patterns. This method allows craftsmen to produce unique and exclusive batik patterns. However, batik businesses must mass produce repeating circular patterns to reach a larger market. Batik businesses want to compose pattern designs and draw them in a way that resembles pendulum oscillations to maintain image reproducibility.

Creating repetitive motifs such as Lissajous patterns has been done using tuning forks [6], oscilloscope, and pendulum [7]. In addition, a visual representation of sounds that form repeating patterns using a harmonograph has also been carried out [8]. Then, repeated Fourier decomposition patterns using ropes were created [9].

As will be done in this research, repetitive motifs can also be done using a pendulum-based machine. The use of pendulums in various studies by Galileo, Huygens, Newton, and Hooke is discussed in detail in [10]. The use of pendulums in art was carried out in [11] to stimulate the creative mind. The trajectory of a spherical pendulum was measured using a simple method [12] using a gravity ball equipped with an LED and a digital camera. Dynamic pendulum analysis on an elliptical trajectory has also been carried out [13]. Furthermore, modeling using pendulums was carried out on oscillations for slewing crane motion [14], pendulum-actuated spherical rolling robots [15], and anti-swing control on overhead cranes [16]. From these studies, this research will focus on creating repeating circular patterns close to the Batik Pendulum pattern currently made using the spherical pendulum trajectory equation approach.

In response to these challenges, a cable-driven parallel robot (CDPR) is proposed to produce Pendulum Batik. CDPR is widely used in various industries, including construction, logistics, motion simulation, production engineering, and entertainment [17]. CDPR offers a slim structure and facilitates accurate positioning [18]. Apart from that, CDPR has three advantages, namely operating in an ample working space, handling large loads, and moving dynamically [17]. This capability makes it easy to simulate pendulum movements. Initial research [19] was carried out to simulate the kinematics of a CDPR end-effector in tracking various patterns and determining the maximum cable length and area required to draw geometric patterns.

This research is part of CDPR's machine and software development project to produce Pendulum Batik, where this research will focus only on software. The proposed CDPR machine has four cables connected to an end-effector, a wax-filled pendulum. CDPR software generates the pattern and sends it to the CDPR machine to apply the wax to the fabric.

CDPR software has three modes: manual, automatic, and gallery. Manual mode allows users to maneuver the CDPR pendulum using the joystick manually. Automatic mode consists of spiral draw, pendulum draw, and execute modes. In spiral draw mode, a spiral pattern is generated, modified, combined, and saved as an image file. Meanwhile, in pendulum draw mode, the parameters are modified to produce a pattern, which will then be saved as an image file. Then, in execute mode, the image file is transferred to the CDPR machine to apply wax to the fabric. Then, gallery mode is for viewing patterns saved in image files.

In the current manual system, mass production of the same pattern is impossible because the trajectory parameters for creating the pattern are unknown. Therefore, software is needed to simplify the mass production process with the same pattern because the patterns created can be reaccessed in gallery mode for further use in execute mode. This research focuses on the pendulum draw mode. The software was created via Android Studio with KOTLIN as the main programming language.

In pendulum draw mode, craftsmen can use a range of parameters to set the pendulum trajectory parameters. The trajectory of the pendulum will form a particular pattern, which is expected to be close to the Batik Pendulum pattern that has currently been made. Research on the trajectory produced by a pendulum has been carried out in [20], [21]. Setting these pendulum parameters is part of the trajectory planning carried out in this research.

## II. Methods

### A. Spherical Pendulum Equation

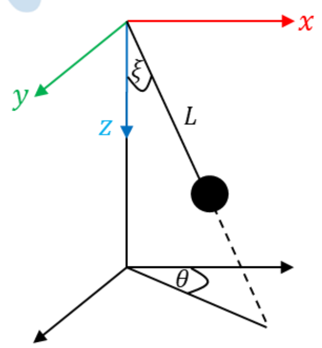



Fig. 2. Spherical pendulum parameter

Fig. 2 illustrates a spherical pendulum with string length $L$, and the angle formed between string $L$ and the z-axis is the polar angle $\xi$, and the angle formed between string $L$ and the x-axis is the azimuthal angle $\theta$. The coordinates of the spherical pendulum's position on each axis can be written as [20], [21]:

$$x_p = L\sin\xi\cos\theta \\ y_p = L\sin\xi\sin\theta \\ z_p = L\cos\xi \tag{1}$$

The speed on each axis can be written as the first derivative of position. The spherical speed of the pendulum can be written as:

$$v_{x,p} = L\dot{\xi}\cos\xi\cos\theta - L\dot{\theta}\sin\xi\sin\theta \\ v_{y,p} = L\dot{\xi}\cos\xi\sin\theta + L\dot{\varphi}\sin\xi\cos\theta \\ v_{z,p} = -L\dot{\xi}\sin\xi \tag{2}$$

Next, to obtain the Lagrange equation $\mathbb{L}$ for the equations of motion $\xi$ and $\theta$, it is necessary to determine the kinetic energy $T$ and potential energy $V$.

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m\left(v_{x,p}^2 + v_{y,p}^2 + v_{z,p}^2\right) = \frac{1}{2}mL^2\left(\dot{\xi}^2 + \dot{\theta}^2\sin^2\xi\right) \tag{3}$$

$$V = -mgz_p = -mgL\cos\xi \tag{4}$$

where $m$ is the pendulum's mass and $g$ is the acceleration due to gravity. So, we get the Lagrange equation $L$ as [20], [21]:

$$\mathbb{L} = T - V = \frac{1}{2}mL^2\left(\dot{\xi}^2 + \dot{\theta}^2\sin^2\xi\right) + mgL\cos\xi \tag{5}$$

In a damped spherical pendulum, the damping effect on the system must be considered through the Rayleigh dissipation function $\mathbb{R}$. [21].

$$\mathbb{R} = \frac{1}{2}\sum b_i v_i^2 = \frac{1}{2}bv^2 = \frac{1}{2}b\left(v_{x,p}^2 + v_{y,p}^2 + v_{z,p}^2\right) = \frac{1}{2}bL^2\left(\dot{\xi}^2 + \dot{\theta}^2\sin^2\xi\right) \tag{6}$$

where $b$ is the damping coefficient.

Therefore, the equation of motion for $\xi$ and $\theta$ can be written as the Euler-Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial\mathbb{L}}{d\dot{\xi}}\right) = \frac{\partial\mathbb{L}}{d\xi} - \frac{\partial\mathbb{R}}{d\dot{\xi}} \\ \ddot{\xi} = \frac{-mg\,sin\,\xi + mL\dot{\theta}^2\,sin\,\xi\,cos\,\xi - bL\dot{\xi}}{mL} \tag{7}$$

$$\frac{d}{dt}\left(\frac{\partial\mathbb{L}}{d\dot{\varphi}}\right) = \frac{\partial\mathbb{L}}{d\varphi} - \frac{\partial\mathbb{R}}{d\dot{\varphi}} \\ m\ddot{\theta}\sin\xi = -2m\dot{\xi}\dot{\theta}\cos\xi - b\dot{\theta}\sin\xi \\ \ddot{\theta} = \frac{-2m\dot{\xi}\dot{\theta}\cos\xi - b\dot{\theta}\sin\xi}{m\sin\xi} \tag{8}$$

## B. Proposed Equation and Parameter

The differential equations (7) and (8) can be solved when the values of $b, m, L, N$, and $t$ are known. Where $N$ is the amount of data, and $t$ is the duration of time the pendulum swings. In addition, to solve the differential equations in (7) and (8), the initial value of the pendulum needs to be known. The values that determine the initial condition of the pendulum from the differential equation are $\xi_0$, $\theta_0$, and $\mathbf{v}_{p,0}$. Here, $\mathbf{v}_{p,0} = \dot{x}_0\hat{\mathbf{i}} + \dot{y}_0\hat{\mathbf{j}} + \dot{z}_0\hat{\mathbf{k}}$. Based on the initial speed value of pendulum, the speed in spherical coordinates can be derived as follows:

$$r = L = \sqrt{x_p^2 + y_p^2 + z_p^2} \\ \xi = \arccos\left(\frac{z_p}{r}\right) \\ \theta = \arctan\left(\frac{y_p}{x_p}\right) \tag{9}$$

Therefore,

$$\mathbf{v}_{p,0} = \frac{x_{p,0}\dot{x}_{p,0} + y_{p,0}\dot{y}_{p,0} + z_{p,0}\dot{z}_{p,0}}{L}\hat{\mathbf{r}} + \frac{\frac{z_{p,0}}{L^2}\left(x_{p,0}\dot{x}_{p,0} + y_{p,0}\dot{y}_{p,0} + z_{p,0}\dot{z}_{p,0}\right) - \dot{z}_{p,0}}{\sqrt{L^2 - z_{p,0}^2}}\hat{\boldsymbol{\xi}} + \frac{\dot{y}_{p,0}x_{p,0} - y_{p,0}\dot{x}_{p,0}}{x_{p,0}^2 + y_{p,0}^2}\hat{\boldsymbol{\theta}} \tag{10}$$

with

$$x_{p,0} = L\sin\xi_0\cos\theta_0 \\ y_{p,0} = L\sin\xi_0\sin\theta_0 \\ z_{p,0} = L\cos\xi_0$$

The spherical pendulum trajectory is obtained on a spherical surface with a radius from the known values and the differential equation. In order to produce a trajectory on the fabric surface, an orthogonal projection of the spherical pendulum trajectory is carried out on the xy-plane by eliminating $z_p$ in the solution coordinates. Hence, the parameters required for planning the spherical pendulum trajectory are $v_{p,0}$, $\theta_0$, $\xi_0$, $N$, $t$, $b$, $m$, and $L$.

## C. Parameter Limits

The lower and upper limits of the spherical pendulum trajectory are defined in equation (11):

$$0 \le v_{p,0} \le 10 \\ 0 \le \theta_0 \le 2\pi \\ 0 \le \xi_0 \le \frac{\pi}{2} \\ 0 \le N \le 10000 \\ 0 \le t \le 120 \\ 0 \le b \le 2 \\ 0{,}1 \le m \le 10 \\ 0{,}1 \le L \le 5 \tag{11}$$

## D. Parameter Limits

After the software is implemented, it is necessary to evaluate its response time. Three response time limits to consider in software applications are as follows [22]:

1. 0.1 seconds is the time limit when the user feels the system reacts instantaneously, which means no other info is necessary except to display the result.

2. 1.0 seconds is the time limit to maintain the user's flow of thought despite noticing a delay. Typically, no info is needed between 0.1 to 1.0 seconds, but the user has lost the feeling of operating directly on the data.
3. Ten seconds is the time limit to maintain the user's attention on the current running activity. For longer delays, users will engage in other activities while waiting for the activity to complete. Therefore, users need to see a progress indicator informing them that the activity is running and the remaining completion time.

## III. Results and Discussion

### A. MATLAB Simulation

The spherical pendulum trajectory simulation used MATLAB's "uifigure" feature. This feature allows simulations to be carried out using the "uislider" to change trajectory parameters. Next, the spherical pendulum trajectory plot can be observed in the "uiaxes" feature. The lower and upper limits of the slider used follow equation (11). The plot has the same size on the x and y axes by looking for the maximum value on both axes. The "uifigure" display that has been designed can be observed in Fig. 3.

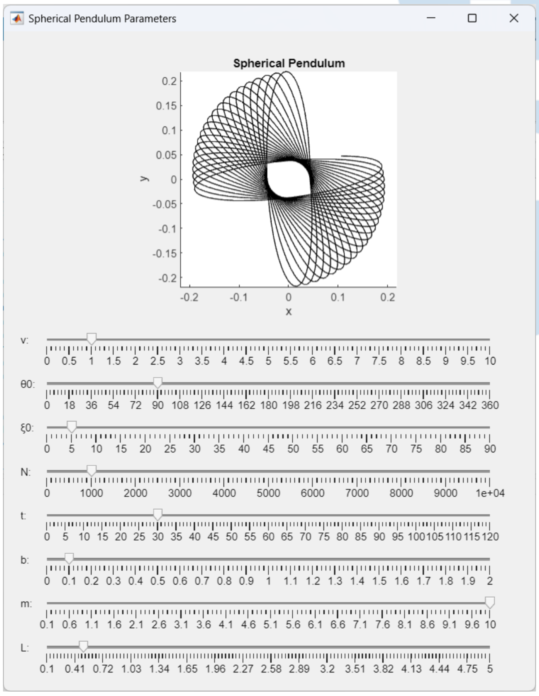
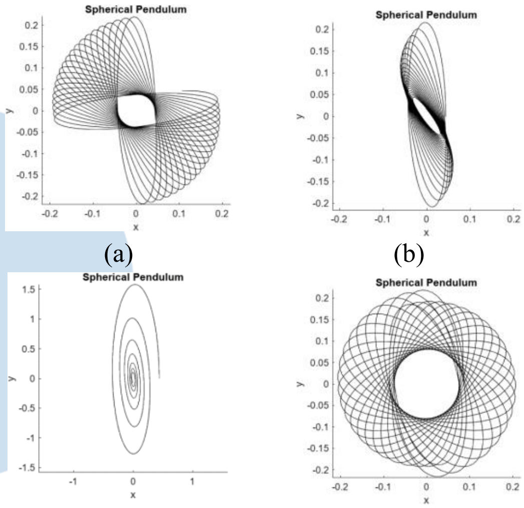



Fig. 3. uifigure MATLAB for spherical pendulum trajectory

Several pendulum parameters were determined to demonstrate the diversity of patterns that can be produced, as shown in Fig. 4 and Table I. Pendulum A is the pendulum trajectory in the default state. Meanwhile, pendulum B, C, and D are various patterns that can be produced by changing the pendulum parameters, as seen in Table I with the yellow highlight. Pendulum pattern B shows a pattern that is often obtained from manual pendulum batik with one string, pendulum pattern C shows a pendulum movement that resembles a spiral, and pendulum pattern D is a new pendulum pattern that cannot be achieved on a manual system but is possible to achieve using the proposed trajectory planning system.

Fig. 4 and Table I show that the parameters proposed for planning the spherical pendulum trajectory can provide various trajectories. Next, these parameters will be used for the spherical pendulum trajectory planning process in the software draw pendulum mode.




(a) (b) (c) (d)

Fig. 4. Spherical pendulum pattern: (a) A, (b) B, (c) C, and (d) D

TABLE I. Spherical Pendulum Parameter

| Pendulum | $v_{p,0}$ | $\theta_0$ | $\xi_0$ | $N$ | $t$ | $b$ | $m$ | $L$ |
|---|---|---|---|---|---|---|---|---|
| A | 1 | 90º | 5º | 1000 | 30 | 0,1 | 10 | 1 |
| B | 1 | 90º | 5º | 1000 | 30 | **1** | 10 | 1 |
| C | **2.5** | 90º | 5º | 1000 | 30 | **2** | 10 | **5** |
| D | 1 | 90º | **10º** | 1000 | 30 | 0.1 | 10 | 1 |


Fig. 5. Pole and fabric size settings


Fig. 6. Pendulum parameters, including linear speed of the pendulum, azimuthal angle, and polar angle


Fig. 7. Pendulum parameters, including the amount of data, duration of time, and damping coefficient


Fig. 8. Pendulum parameters, including damping coefficient, mass of the pendulum, and length of the string


Fig. 9. The trajectory results from changing the pendulum parameters

### B. *Software Implementation*

The procedure for generating spherical pendulum parameters in the implemented software is illustrated in Fig. 5 – Fig. 9. In pendulum draw mode, the user must define the pole size and fabric used. Next, the user can change the pendulum parameters by moving the slider on the right side of the screen. The resulting trajectory is displayed on the left side of the screen. User can save patterns in the gallery for later use.

The required time since the parameters are changed until the path is generated is 1 – 2 seconds. Because the required time is less than 10 seconds, it is within the user's flow of thought, and therefore, there is no need to show a progress indicator to the user.

## IV. Conclusions

A trajectory planning algorithm that accepts parameter values has been designed to produce a spherical pendulum pattern. The parameters consist of $v_{p,0}$, $\theta_0$, $\xi_0, N, t, b, m,$ and $L$. Modifying these eight inputs between the specified lower and upper limits provides a variety of spherical pendulum patterns.

The spherical pendulum trajectory planning in the implemented software requires 1-2 seconds since the parameters are changed until the trajectory is produced. Because the response time is less than 10 seconds, there is no need to show a progress indicator to the user.

For future work, the trajectory generated from the draw pendulum mode can be saved into gallery mode for later use in the execute mode. Therefore, the user can use the saved pendulum pattern and send it to the machine to apply the wax to the fabric.

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